4) $\boxed{3}\cdot\boxed{1}+\boxed{3}\cdot\boxed{2}+\boxed{3}\cdot\boxed{3}+\cdots\cdots+\boxed{3}\cdot\boxed{20}$

$\frac{12}{4}$

K: 1 2 3 . . . . 20

$$\sum_{K=1}^{20} 3K$$

12) $$\sum_{K=1}^{100}(7K+1) = \sum_{K=1}^{100} 7K + \sum_{K=1}^{100} 1$$

$$\sum_{K=1}^{100} 7K = 7\sum_{K=1}^{100} K$$

p396

$$= 7\cdot\frac{100(101)}{2}$$

$$\sum_{K=1}^{100} 1 = 1+1+1+\cdots 1$$

$$= 100$$

13, 15, 17, 19, 23, 24

(13) $\sum_{k=1}^{20} k^2 =$

$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

$\frac{20(21)(41)}{6}$

$(10)(7)(41)$

$= 2870$

(15) $\sum_{k=1}^{30} k(k-2)(k+2)$

$= \sum_{k=1}^{30} k^3 - 4k$

$= \sum_{k=1}^{30} k^3 - 4\sum_{k=1}^{30} k$

$\left(\frac{(30)(31)}{2}\right)^2 - 4\left(\frac{(30)(31)}{2}\right)$

$465^2 - (60)(31)$

$465^2 - 1860$

$(15)(15)(31)^2 - (60)(31)$

$$\begin{array}{r} 216225 \\ -\ 1860 \\ \hline 214365 \end{array}$$

17) $\sum_{K=1}^{n} \frac{3K}{n} = \frac{3}{n}\sum_{K=1}^{n} K$

$= \frac{3}{n}\left(\frac{n(n+1)}{2}\right)$

$= \frac{3(n+1)}{2}$

19) $\sum_{K=1}^{n-1} \frac{K^3}{n^2} = \frac{1}{n^2}\sum_{K=1}^{n-1} K^3$

$= \frac{1}{n^2}\left[\frac{(n-1)((n-1)+1)}{2}\right]^2$

$\sum_{K=1}^{n} K^3 = \left(\frac{n(n+1)}{2}\right)^2$

$\frac{1}{n^2}\left(\frac{(n-1)(n)}{2}\right)^2 = \frac{(n-1)^2}{4}$

23) $$\lim_{n\to+\infty} \frac{1+2+3+\dots+n}{n^2}$$

$$= \lim_{n\to\infty} \frac{1}{n^2}\left(\sum_{k=1}^{n} k\right)$$

$$= \lim_{n\to\infty} \frac{1}{n^2}\left(\frac{n(n+1)}{2}\right)$$

$$= \lim_{n\to\infty} \frac{n^2+n}{2n^2} =$$

$$\lim_{n\to\infty} \frac{n^2\left(1+\frac{1}{n}\right)}{n^2(2)} = \frac{1}{2}$$

24) $$\lim_{n\to\infty} \frac{1^2+2^2+\dots+n^2}{n^3}$$

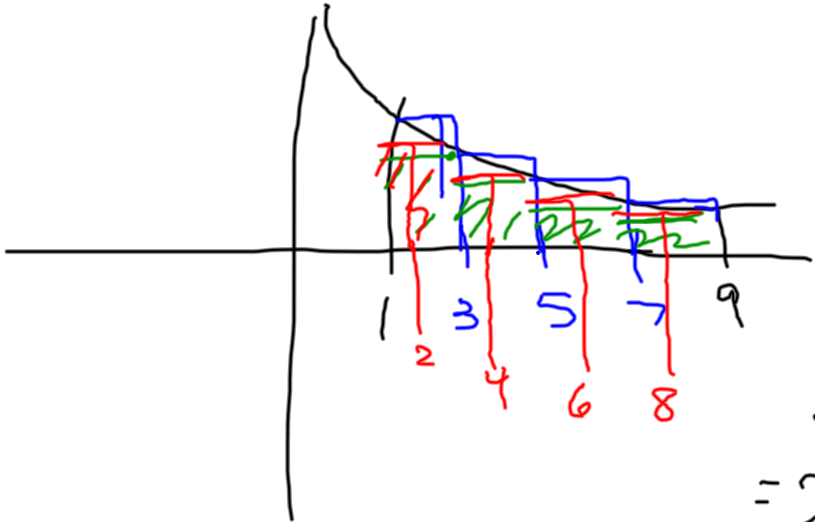
$$= \lim_{n\to\infty} \frac{1}{n^3}\left(\sum_{k=1}^{n} k^2\right)$$

$$= \lim_{n\to\infty} \frac{1}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$$

$$= \dots\dots = \frac{1}{3}$$

30
38-39
44-45

30) $f(x)=\frac{1}{x}$; $[1,9]$




L.e.a.

$2\cdot f(1)+2\cdot f(3)+2\cdot f(5)+2\cdot f(7)$

$2\left[\frac{1}{1}+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}\right]$

R.e.a.

$2\cdot f(3)+2\cdot f(5)+2\cdot f(7)+2\cdot f(9)$

$=2\left[\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\frac{1}{9}\right]$

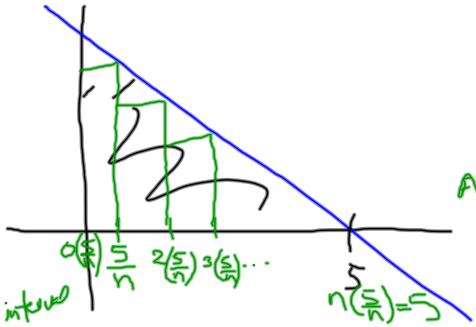
Mpa

$2\cdot f(2)+2\cdot f(4)+2f(6)+2f(8)$

$=2\left[\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}\right]$

38) $y = 5 - x \; ; \; [0,5]$

$$\text{Area} = \lim_{n\to\infty} \sum_{k=1}^{n} \Delta x_k \, f(x_k^*)$$




width interval

$5 - 0 = 5$

divide into 'n' pieces, each with a width of $\frac{5}{n}$

Area thinking $= \left(\frac{5}{n}\right) \cdot f\left(1\left(\frac{5}{n}\right)\right) +$

$\left(\frac{5}{n}\right) \cdot f\left(2\left(\frac{5}{n}\right)\right) +$

$\left(\frac{5}{n}\right) \cdot f\left(3\left(\frac{5}{n}\right)\right) + \ldots$

$\vdots$

$\left(\frac{5}{n}\right) \cdot f\left(n\left(\frac{5}{n}\right)\right)$

$$\sum_{k=1}^{n} \left(\frac{5}{n}\right) f\left(k\left(\frac{5}{n}\right)\right)$$

$$= \sum_{k=1}^{n} \left(\frac{5}{n}\right)\left(5 - \frac{5k}{n}\right)$$

$$= \sum_{k=1}^{n} \left(\frac{25}{n} - \frac{25k}{n^2}\right)$$

$$= 25 - \frac{25}{n^2} \sum_{k=1}^{n} k$$

$$= 25 - \frac{25}{n^2}\left(\frac{n(n+1)}{2}\right)$$

Area:

$$\lim_{n\to\infty} 25 - \frac{25}{2}\left(\frac{n(n+1)}{n^2}\right)$$

$$= 25 - \frac{25}{2} = 12.5$$

39 $y = 9 - x^2$; $[0, 3]$


Area =

$$\underbrace{\lim_{n\to\infty}}_{\text{limit}} \underbrace{\sum_{k=1}^{n}}_{\text{summation}} \underbrace{\Delta x_k \cdot f(x_k^*)}_{\substack{\text{area of rectangle}\\ \text{base} \cdot \text{height}}}$$

① width of entire region $= 3 - 0 = 3$

② divide into $n$ pieces each w/ a width $= \frac{3}{n}$

③ x values $0 \to 3$

$0,\ 0+\frac{3}{n},\ 0+2\left(\frac{3}{n}\right),\ 0+3\left(\frac{3}{n}\right), \ldots, 0+n\left(\frac{3}{n}\right) = 3$

④ pick $x_k^*$

39: pick right endpt:

$0+\frac{3}{n},\ 0+2\left(\frac{3}{n}\right),\ 0+3\left(\frac{3}{n}\right), \ldots$

⑤ areas of rect?

$\left(\frac{3}{n}\right) f\left(0+\frac{3}{n}\right) + \left(\frac{3}{n}\right) f\left(0+2\left(\frac{3}{n}\right)\right) + \ldots$

⑥ $\Sigma$ notation

$y = 9 - x^2 \longrightarrow$

$\left(\frac{3}{n}\right)\left(9 - \left(1\left(\frac{3}{n}\right)\right)^2\right) + \left(\frac{3}{n}\right)\left(9 - \left[2\left(\frac{3}{n}\right)\right]^2\right) + \ldots$

$$\sum_{k=1}^{n} \left(\frac{3}{n}\right)\left(9 - \left(k\left(\frac{3}{n}\right)\right)^2\right)$$

⑦ $\lim_{n\to\infty}$ $\boxed{\phantom{xx}}$

the only ∠ worth finding is a triangle :)

1c) $\sum_{i=-4}^{1}(i^2-i) = 20 + 12 + 6 + 2 + 0 + 0 = 40$

$(-4)^2-(-4)$ $\quad (-3)^2-(-3)$ $\quad (-2)^2-(-2)$ $\quad \underline{(-1)^2-(-1)}$

$= \sum_{i=-4}^{1} i^2 - \sum_{i=-4}^{1} i$

p396

$$\sum_{i=1}^{6}(i-5)^2 - \sum_{i=1}^{6}(i-5)$$

$$\sum_{i=1}^{6} i^2 - 10\sum_{i=1}^{6} i + \sum_{i=1}^{6} 25 - \sum_{i=1}^{6} i + \sum_{i=1}^{6} 5$$

1b) $\sum_{j=2}^{6}(3j-1) = 5+8+11+14+17 = 55$

$\Downarrow$

$\sum_{j=2}^{6} 3j - \sum_{j=2}^{6} 1$

book

$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

$2+3+4+5+6$ $\qquad$ $1+2+3+4+5+6$

$3\sum_{j=2}^{6} j = 3\left[\sum_{j=1}^{6} j - 1\right]$

$\sum_{j=1}^{5}(j+1)$

$= 3\left(\frac{n(n+1)}{2} - 1\right)$

$\sum_{j=1}^{5} j + \sum_{j=1}^{5} 1$

$\frac{5\cdot 6}{2} + 5$

$= 3\left(\frac{6\cdot 7}{2} - 1\right) = 60 - 5 = 55$

$\sum_{j=2}^{6} 1 =$

$1+1+1+1+1$

$= 5$

12) $\sum_{k=1}^{100} 7k+1 = 7\sum_{k=1}^{100} k + \sum_{k=1}^{100} 1$

$= 7\left(\frac{(100)(101)}{2}\right) + 100$

$= 7 \cdot 5050 + 100$

$= 35450$

17) $\sum_{k=1}^{n} \frac{3k}{n}$

$= \frac{3}{n}\sum_{k=1}^{n} k$

$= \frac{3}{n}\left(\frac{n(n+1)}{2}\right)$

$= \frac{3(n+1)}{2}$

19) $\sum_{k=1}^{n-1} \frac{k^3}{n^2}$

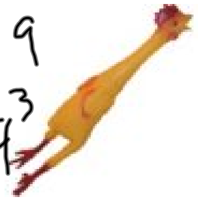
$= \frac{1}{n^2}\sum_{k=1}^{n-1} k^3$

$= \frac{1}{n^2}\left(\frac{(n-1)((n-1)+1)}{2}\right)^2$

$= \frac{1}{n^2}\left(\frac{(n-1)(n)}{2}\right)^2 = \frac{(n-1)^2}{4}$

$\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$

17, 19, 23, 24



23)

$$\lim_{n\to\infty} \frac{1+2+3+\ldots\ldots+n}{n^2}$$

$$= \lim_{n\to\infty} \frac{1}{n^2}\left(\sum_{k=1}^{n} k\right)$$

$$= \lim_{n\to\infty} \frac{1}{n^2}\left(\frac{n(n+1)}{2}\right)$$

$$= \lim_{n\to\infty} \frac{n+1}{2n} =$$

$$\lim_{n\to\infty} \frac{n(1+\frac{1}{n})}{n(2)} = \frac{1}{2}$$

24)

$$\lim_{n\to\infty} \frac{1^2+2^2+3^2+\ldots+n^2}{n^3}$$

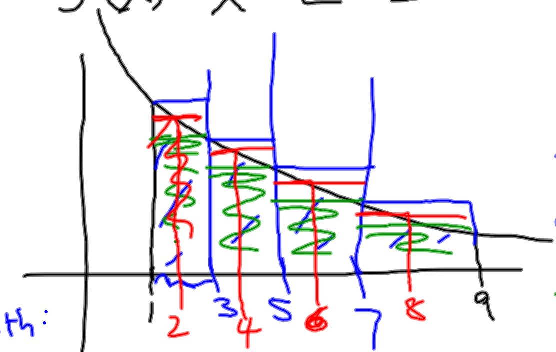
$$= \lim_{n\to\infty} \frac{1}{n^3} \sum_{k=1}^{n} k^2$$

$$= \lim_{n\to\infty} \frac{1}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$$

$$= \lim_{n\to\infty} \frac{2n^2+3n+1}{6n^2}$$

$$= \lim_{n\to\infty} \frac{n^2(2+\frac{3}{n}+\frac{1}{n^2})}{n^2(6)} = \frac{2}{6} = \frac{1}{3}$$

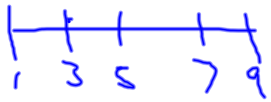
30) $f(x)=\frac{1}{x}$; $[1, 9]$




width: $9-1=8$

each of the 4 sub-intervals

$\frac{8}{4}=2$




L.e.a.

$2\cdot f(1)+2\cdot f(3)+2\cdot f(5)+2f(7)$

$=2\left[\frac{1}{1}+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}\right]$

R.e.a.

$2\cdot f(3)+2f(5)+2f(7)+2f(9)$

$=2\left[\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\frac{1}{9}\right]$

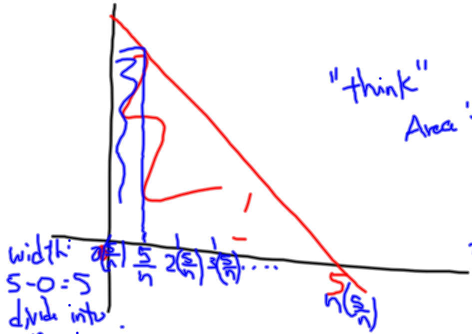
M.p.a.

$=2f(2)+2f(4)+2f(6)+2f(8)$

$=2\left[\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}\right]$

38) $y = 5 - x \; ; [0, 5]$

Area $= \lim_{n\to\infty} \sum_{k=1}^{n} \Delta x_k \cdot f(x_k^*)$




"think"

Area: $\left(\frac{5}{n}\right)\left(f\left(\frac{5}{n}\right)\right) + \left(\frac{5}{n}\right) f\left(2\left(\frac{5}{n}\right)\right) + \left(\frac{5}{n}\right) f\left(3\left(\frac{5}{n}\right)\right) + \dots + \frac{5}{n} f\left(n\left(\frac{5}{n}\right)\right)$

$\left(\frac{5}{n}\right)\left(5 - 1\left(\frac{5}{n}\right)\right) + \left(\frac{5}{n}\right)\left(5 - 2\left(\frac{5}{n}\right)\right) + \dots \left(\frac{5}{n}\right)\left(5 - n\left(\frac{5}{n}\right)\right)$

Area: $\sum_{k=1}^{n} \left(\frac{5}{n}\right)\left(5 - k\left(\frac{5}{n}\right)\right)$

$44 \sum_{k=1}^{n} \left(\frac{5}{n}\right)\left(5 - (k-1)\frac{5}{n}\right)$

$= \sum_{k=1}^{n} \left(\frac{25}{n} - k\frac{25}{n^2}\right)$

$= \frac{25}{n}\sum_{k=1}^{n} 1 - \frac{25}{n^2}\sum_{k=1}^{n} k$

$= \frac{25}{n}(n) - \frac{25}{n^2}\left(\frac{n(n+1)}{2}\right)$

44

$\lim_{n\to\infty} 25 - \frac{25}{2}\left(\frac{n(n+1)}{n^2}\right)$

$= 25 - \frac{25}{2} \lim_{n\to\infty} \frac{n(n+1)}{n^2}$

$= 25 - \frac{25}{2}(1) = 25 - \frac{25}{2} = \frac{25}{2}$

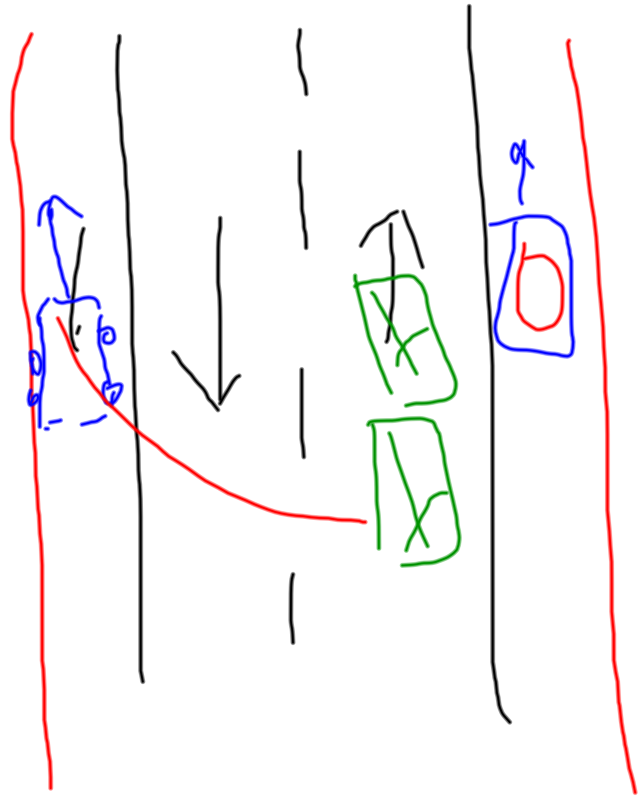
$\lim_{n\to\infty} \frac{n+1}{n} = \lim_{n\to\infty}\left(1 + \frac{1}{n}\right)$

$\sum_{k=1}^{n} \frac{25}{n} - \frac{25}{n^2}(k) + \frac{25}{n^2}$

$= 25 - \frac{25}{n^2}\left(\frac{n(n+1)}{2}\right) + \frac{25}{n}$

$\lim_{n\to\infty} 25 - \frac{25}{2}\frac{n(n+1)}{n^2} + \frac{25}{n}$

$= 25 - \frac{25}{2} + 0 = 12.5$

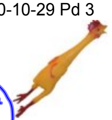


6.4 homework

2010-10-29 Pd 3

39] $y = 9 - x^2$; $[0, 3]$


Area =



$$\underbrace{\lim_{n\to\infty}}_{\text{lim}} \underbrace{\sum_{k=1}^{n}}_{\text{add em up}} \underbrace{\underbrace{\Delta x_k}_{\text{base}} \cdot \underbrace{f(x_k^*)}_{\text{height}}}_{\text{area of rectangle}}$$

① width of interval $= 3 - 0 = 3$

② divide into $n$ pieces each of length $\frac{3}{n}$

③ find the x values that separate the subintervals

$0, 0+1\left(\frac{3}{n}\right), 0+2\left(\frac{3}{n}\right), 0+3\left(\frac{3}{n}\right), \ldots 0+n\left(\frac{3}{n}\right)$

④ identify the $x_k^*$ (39: Right endpt)

$0+1\left(\frac{3}{n}\right), 0+2\left(\frac{3}{n}\right) \ldots$

⑤ develop $\Sigma$ $\quad f\left(\frac{3k}{n}\right)$

$$\sum_{k=1}^{n}\left(\frac{3}{n}\right)\left(f\left(k\left(\frac{3}{n}\right)\right)\right)$$

need closed form

$$\lim_{n\to\infty} \sum_{k=1}^{n}\left(\frac{3}{n}\right)\left(9 - \left(k\left(\frac{3}{n}\right)\right)^2\right)$$

⑥

$\sum_{k=1}^{n} 1 = n$

$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

$\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$